2/20/19 The integrated rate laws: [ ] vs. t

1^{st} order

first order.

given: $A \longrightarrow$ Products , rate $= K[A]^1$

$$-\frac{\Delta[A]}{\Delta t} = K[A]^1$$ (differential rate law)

calculus

$$\ln[A]_t = -Kt + \ln[A]_0$$ (integrated rate law)

natural logarithm — molar conc of A @ time t — time — rate constant — molar conc of A @ time 0

can rearrange using props of logs ($\log A - \log B = \log \frac{A}{B}$)

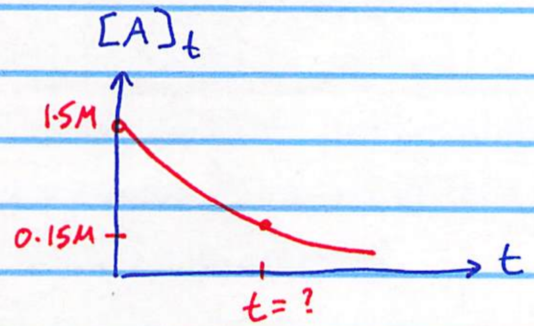
$$\boxed{\ln\left(\frac{[A]_t}{[A]_0}\right) = -Kt} \xrightarrow[e^{\wedge}]{\text{anti-ln}} \frac{[A]_t}{[A]_0} = e^{-Kt}$$

(exponential decay)

ex: 1^{st} order rxn: $A \rightarrow P$

$\boxed{K = 0.038\,s^{-1}}$

Q: How long does it take for an init conc of A (1.5M) [A]₀ to become a final conc of 0.15M? [A]ₜ



$$\ln[A]_t = -Kt + \ln[A]_0$$

$$\frac{\ln[A]_t - \ln(A)_0}{-K} = \frac{-\cancel{K}t}{-\cancel{K}}$$

$$t = \frac{\ln[A]_t - \ln[A]_0}{-K} = \frac{\ln(0.15) - \ln(1.5)}{-0.038\,s^{-1}}$$

$= 61\,s$

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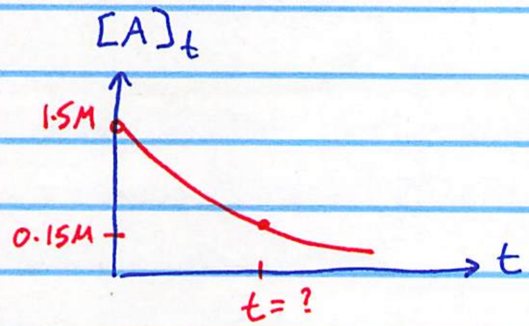

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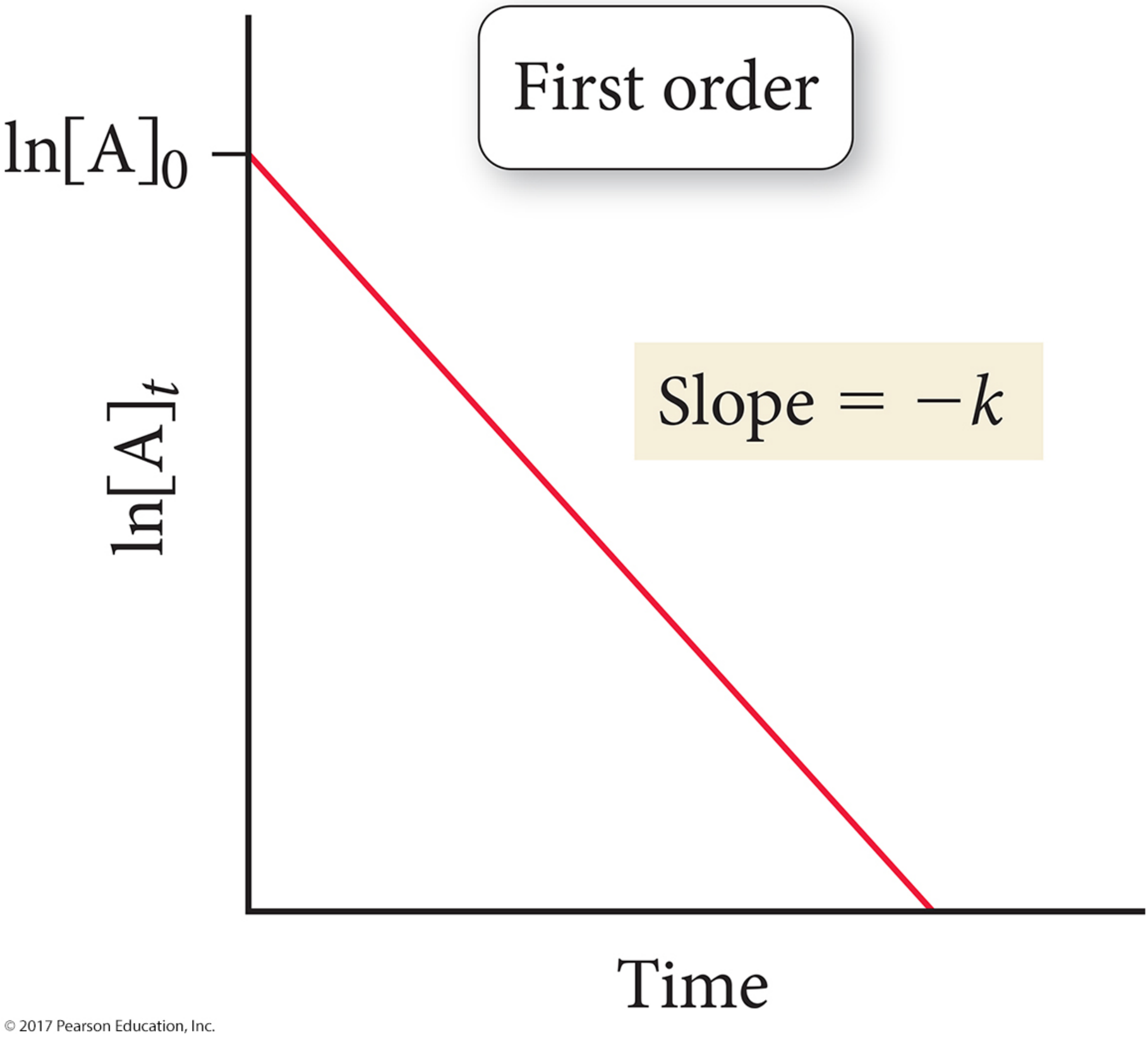

First order
ln[A]$_0$
ln[A]$_t$
Slope = −k
Time

$$\frac{\ln\left(\frac{[A]_t}{[A]_0}\right)}{-k} = \frac{\cancel{-k}t}{\cancel{-k}}$$

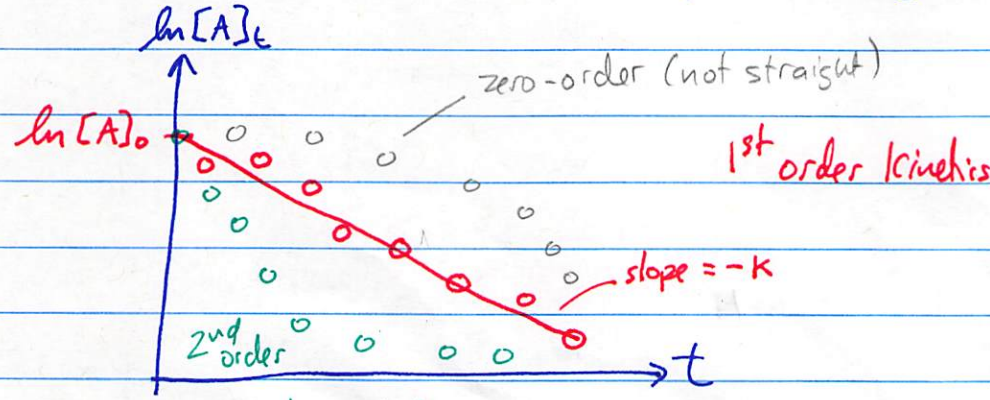
$$\Rightarrow t = \frac{\ln\left(\frac{0.15M}{1.5M}\right)}{-0.038 s^{-1}} = 61 s$$

Since: $\ln[A]_t = \underbrace{-k}_{\text{rate const.}} t + \underbrace{\ln[A]_0}_{\text{orig conc}}$

$$y = mx + b$$

let's plot $\ln[A]_t$ vs. t

- IF it is 1^{st} order in A, then: straight line!



straight line (x)

2^{nd} order kinetics

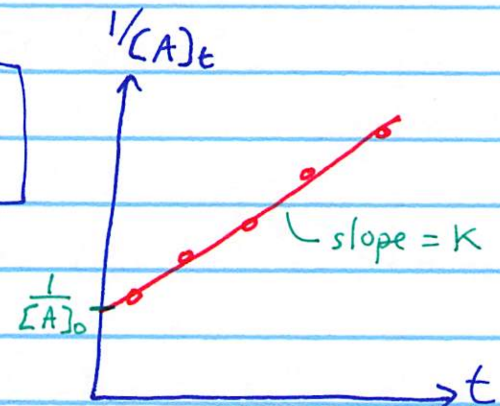
$$A \rightarrow P \quad , \quad rate = K[A]^2$$

$$-\frac{\Delta[A]}{\Delta t} = K[A]^2$$

calculus 2

$$\boxed{\frac{1}{[A]_t} = Kt + \frac{1}{[A]_0}}$$

$$y = mx + b$$



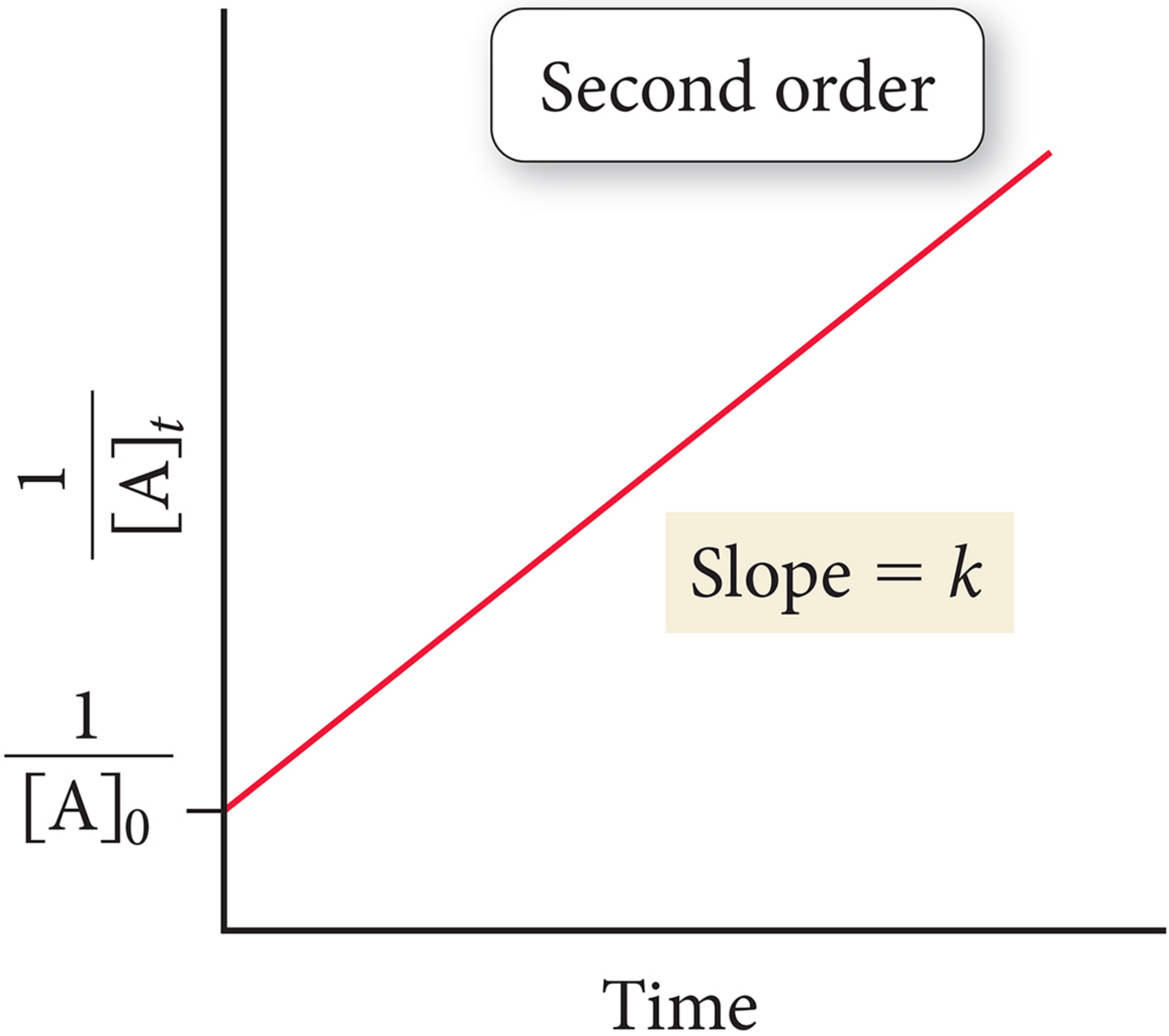
Ex: What's conc of A, after 3.0 min, if orig conc was 0.10M and $K = 0.30\,M^{-1}s^{-1}$ (2nd order kinetics)

$[A]_t$? $t = 180.0s$ $[A]_0$ K

$$\frac{1}{[A]_t} = Kt + \frac{1}{[A]_0}$$

$$\Rightarrow [A]_t = \frac{1}{0.30\,M^{-1}\cancel{s^{-1}} \times 180\cancel{s} + \frac{1}{0.10M}} \qquad \frac{1}{1/M}$$

$$= 0.016\,M$$


Second order
Slope = k
1/[A]_t
1/[A]_0
Time